IMPACT: International Journal of Information Engineering and Technology (IMPACT: IJIET)
ISSN(E): Applied; ISSN(P): Applied
Vol. 1, Issue 1, Jun 2016, 1-8

CONTROL THE CHAOTIC RIKITAKE SYSTEM BY PID CONTROLLER

MOJTABA CHAVOOSHI ZADE

Department of Electrical Engineering, Dariun Branch, Islamic Azad University, Dariun, Iran

ABSTRACT

In this paper a PID controller for the Chaos Rikitake system is introduced. The mathematical model of the Rikitake system consists of three nonlinear differential equations, which are found to be the same as the mathematical model of the well known Lorenz system. The study showed that the system is experiencing a chaotic behavior at certain value of the control parameter. The experienced chaotic oscillations may simulate the reversal of the Earth's magnetic field. The Proportional - Integral - Derivative (PID) controller is one of the most popular controllers used in industry because of their remarkable effectiveness, simplicity of implementation and broad applicability. In this paper the behavior of chaotic Rikitake system is investigated, after that a PID controller is implemented to achieve the stability of system. Simulation results illustrate the effectiveness and validity of the proposed approach.

KEYWORDS: Chaos, Rikitake System, Sinusoid, Stability, PID Controller

INTRODUCTION

Many theories have been advanced to explain the origin of the earth's main dipole field, the Rikitake system introduced for describing the irregular polarity switching of the Earth's magnetic field. But intervals among such geomagnetic polarity reversals are highly irregular. Thus while their average is about 3×10^5 years, there are intervals as long as 3×10^7 years without polarity change. From introduce Rikitake system, till now many control approaches has been presented. At the bellow we will review this works together:

Pecorra and Carroll [1], Ott.E, et.al.[2], Carlos Aguilar-Ibañez, et.al. [3], Mohammad Ali Khan [4] Synchronization for chaotic system has been investigated. In the last years, some methods to achieve synchronization have been proposed from the control theory perspective such as the famous observer-based approach [5], [6], and the so-called adaptive synchronization method [7]. Two research directions have been already conformed in synchronizing chaos: (i) analysis and (ii) synthesis. Analysis problem comprises: (a) the classification of synchronization phenomena [8], (b) the construction of a general framework for unifying chaotic synchronization [9], and (c) the comprehension of the synchronization properties, for instance, robustness [10] or geometry [11]. Liu Xiao-Jun, et.al. [12] analyzed the dynamics of Rikitake two-disk dynamo to explain the reversals of the Earth's magnetic field. They concluded that the chaotic behavior of the system can be used to simulate the reversals of the geomagnetic field. The Rikitake chaotic attractor was studied by several authors. T. McMillen [13] and Mohammad Javidi et.al.[14] has studied the shape and dynamics of the Rikitake attractor. J. Llibre .et.al [15] used the Poincare compactification to study the dynamics of the Rikitake system at infinity. Chien- Chih Chen et.al [16] have studied the stochastic resonance in the periodically forced Rikitake dynamo. In the past decade, many researchers start working on controlling the chaotic behaviors. Harb and Harb [17] have designed a nonlinear controller to control the chaotic behavior in the phase-locked loop by means of nonlinear control. Ahmad Harb[18] have designed a controller to control the unstable chaotic oscillations by means of back stepping method. U.E. Vincent, R. Guo [19], Park et.al[20] and Jeong et.al[21] They have presented a controller by use of adaptive method and controlled chaotic Rikitake system.

In this paper we want to control chaotic Rikitake system by use of Takagi–Sugeno (T-S) fuzzy model [22] has attracted a great deal of attention. The main purpose of the T-S fuzzy model is to represent or approximate a complex nonlinear system. The T-S fuzzy model approach will provide a powerful method for analysis of nonlinear systems [23, 24]. After that we will optimize the fuzzy designed controller by use of GA(Genetic Algorithm) technique[25, 26, 27], and monitor operation of final controller on chaos Rikitake system.

Description Mathematical of Rikitake System

The Rikitake system consists of two conducting rotating disks see Figure 1. These disks are connected into two coils. The current in each coil feeds the magnetic field of the other. The self inductance (L) and resistance (*R*) are the same in each circuit. An external constant mechanical torque (G) for each circuit is applied on the axis to rotate with an angular velocity [28].

The original differential equations derived by Rikitake are:

$$\begin{cases} L_1 \dfrac{dI_1}{dt} + R_1 I_1 = \omega_1 M I_2 \\ L_2 \dfrac{dI_2}{dt} + R_2 I_2 = \omega_2 N I_1 \\ C_1 \dfrac{d\omega_1}{dt} = G_1 - M I_1 I_2 \\ C_2 \dfrac{d\omega_2}{dt} = G_2 - N I_1 I_2 \end{cases} \tag{1}$$

where L, R are the self-inductance and resistance of the coil, the electric currents, I, ω, C, G are the electric currents, the angular velocity , momentum of inertia, and the driving force; M, N are the mutual inductance between the coils and the disks.

Now we consider a further simplification by

$L_1=L_2$, $R_1=R_2$, $M=N$, $C_1=C_2$, $G_1=G_2$

and set:

$$I_1 = \sqrt{\frac{G}{M}}x, \quad I_2 = \sqrt{\frac{G}{M}}y, \quad \omega_1 = \sqrt{\frac{GL}{CM}}z \tag{2}$$

$$\omega_2 = \sqrt{\frac{GM}{CM}}(z-a), \quad t = \sqrt{\frac{CL}{GM}}t', \quad u = R\sqrt{\frac{C}{LGM}} \tag{3}$$

Where, constant parameter a, u>0.

The system mathematical model can be written as follows:

$$\begin{cases} x_1' = -ux_1 + x_2x_3 \\ x_2' = -ux_2 + (x_3 - a)x_1 \\ x_3' = 1 - x_1x_2 \end{cases} \tag{4}$$

where (x, y, z) ∈ R3 are the state variables and a > 0, u > 0 are parameters. Note that system (4) is a quadratic system in R^3. The choice of the parameters a > 0 and u > 0 reflects a physical meaning in the Rikitake model. For study physical meaning can see [30]. Here we suppose a=5 & u=2, we know according to ref. [28] this system in some values is unstable and we choose this system in chaotic mode.

Note that x and y are corresponding to the electric currents, while z is corresponding to the angular velocity. For more details read ref. [29, 30]. At the bellow Figure 1 we see the shape of Rikitake and in Figure 2 and Figure 3 behavior of system that it is chaotic behavior.

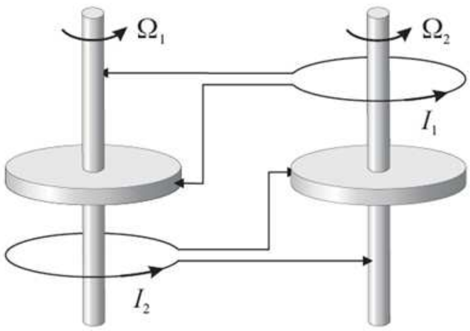

Figure 1: The Rikitake Dynamo is Composed of Two Disk Dynamos Coupled To Another

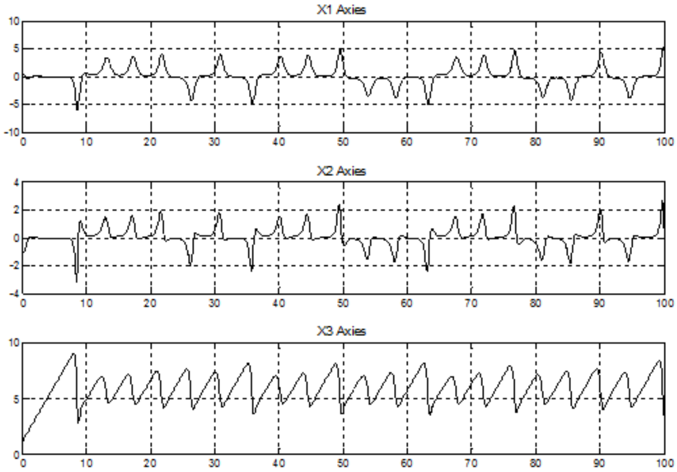

Figure 2: Behavior of Rikitake System without Controller

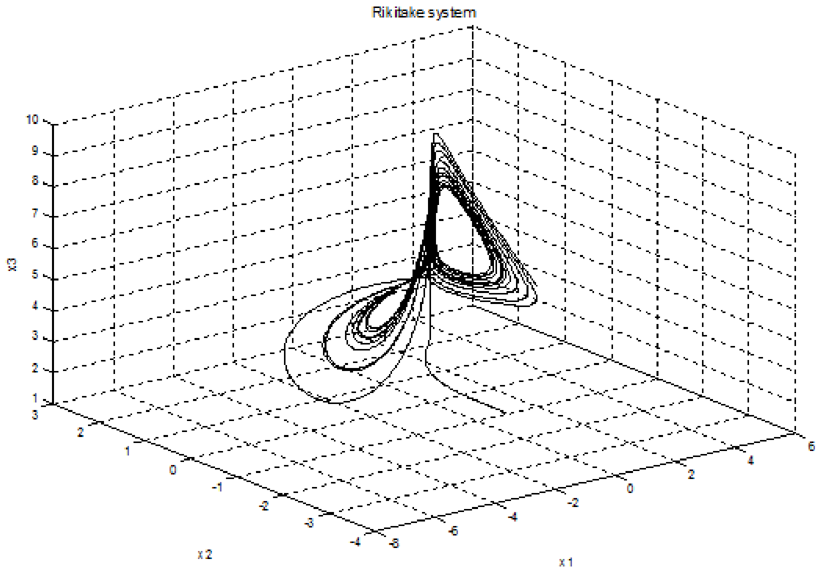

Figure 3: Behavior of Rikitake System in 3D Plot

RESULTS AND DISCUSSIONS

In this section of paper we want to explain the approach of control by use of applying PID controller on chaotic Rikitake system and will monitor behavior of system and effect of PID controller.

The PID controllers have been at the heart of control engineering practice over the last decades. They are widely used in industrial applications as no other controllers match the simplicity, clear functionality, applicability and ease of use. The PID controllers was introduced in 1910 and their use and popularity had grown particularly after the Ziegler–Nichols empirical tuning rules in 1942 (Ziegler and Nichols, 1942). This control approach is an online and proven method however it requires experiences and very aggressive tuning for the process [31].

A PID controller calculates an error value as the difference between a measured process variable and a desired setpoint. The controller attempts to minimize the error by adjusting the process through use of a manipulated variable. The form of the PID algorithm is:

$$u(t)=k_p e(t)+k_i\int_0^t e(\tau)d\tau+k_d\frac{d}{dt}e(t) \quad (5)$$

K_p: Proportional gain

K_i: Integral gain

K_d: Derivative gain

e: Error

Before apply PID controller we should adjust the values of P,I and D we suppose these values: P=30 , I=300, D=30, after that will apply PID controller on chaotic Rikitake system. At the bellow Figure 4 we see that the PID controller controlled very fast with a little over shooting at the Figure 5 behavior of controlled Rikitake system in 3D plot has been drawn and in Figure 6 approach of applying PID controller on Rikitake system has been shown. Note that this system is intrinsically chaos [32] but when we applied PID controller on it this system be controlled.

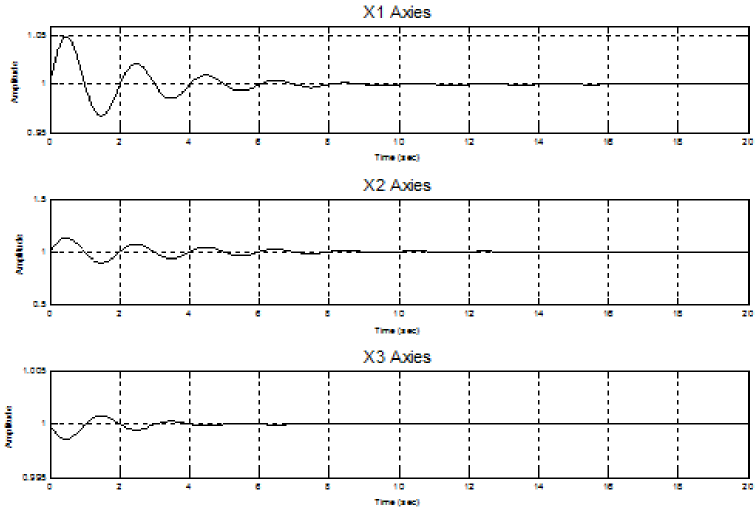

Figure 4: Behavior of System after Control

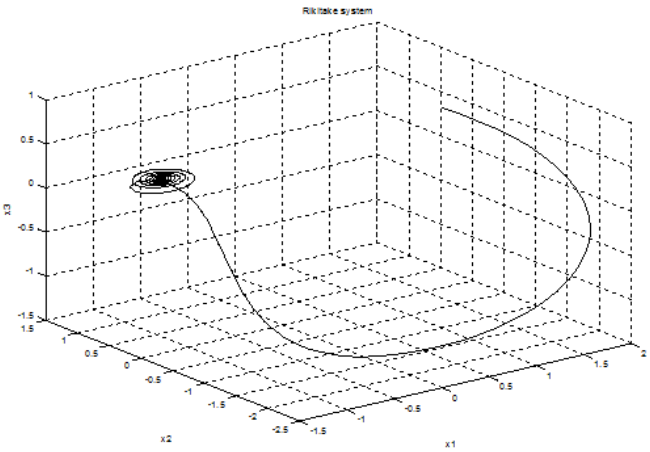

Figure 5: Behavior of Controlled Rikitake System in 3D Plot

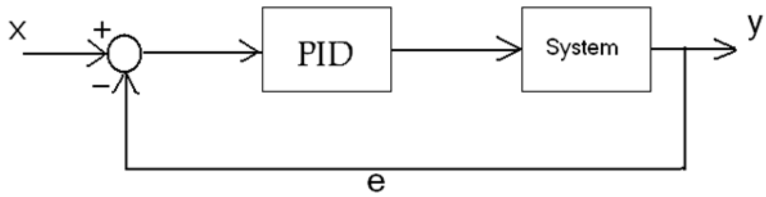

Figure 6: Block Diagram of PID Controller

CONCLUSIONS

In this paper the Rikitake chaotic system has been investigated and its behavior has been studied. The study showed that the system has intrinsic chaotic behavior at certain value. PID controller was suggested to control the system; the designed controller was so effective to eliminate the unstable chaotic oscillations of the Rikitake two-disk system.

ACKNOWLEDGEMENTS

We acknowledge our friend, Mehdi fatemi, and the Associate Editor and anonymous reviewers for their valuable comments and suggestions that have helped us to improving the paper.

REFERENCES

1. Pecora, L.M., Carroll, T.L.: Synchronization in chaotic systems. Phys. Rev. Lett. 64, 821–825, 1990.
2. E. Ott, C. Grebogi and J.A. Yorke, Controlling chaos, Phys. Rev. Lett., 64, 1196-1199, 1990.
3. Carlos Aguilar-Ibañez, Rafael Martinez-Guerra,Ricardo Aguilar-López, Juan L. Mata-Machuca, Synchronization and parameter estimations of an uncertain Rikitake system, Physics Letters A 374, 3625–3628, 2010.
4. Mohammad Ali Khan,Different Synchronization Schemes for chaotic Rikitake Systems ,Journal of Advanced Computer Science and Technology, 1 (3), 167-175, 2012.
5. Morgul O, Solak E, Observed based synchronization of chaotic systems, Phys Rev E, 54, 4803-4811, 1996.
6. Wen Yu, High-gain Observer for chaotic synchronization and secure communication, International Journal Bifurcation and Chaos. 18, 487-500, 2005.
7. R. Femat, J. Alvarez-Ram´ırez, G. Fern´andez-Anaya, Adaptive synchronization of high-order chaotic systems: a feedback with low-order parametrization, Phisica D. 139, 231-246, 2000.
8. Femat, R., Solis-Perales, G.: On the chaos synchronization phenomena. Phys. Lett. A 262, 50–60, 1999.
9. Park, J.H., Ji, D.H., Won, S.C., Lee, S.M.: Adaptive $H\infty$ synchronization of unified chaotic systems. Mod. Phys. Lett. B 23, 1157–1169, 2009.
10. Huang, H., Feng, G., Sun, Y.: Robust synchronization of chaotic systems subject to parameter uncertainties. Chaos 19, 033128, 2009.
11. Yajima, T., Nagahama, H.: Geometrical unified theory of Rikitake system and KCC-theory. Nonlinear Anal. 71, e203–e210, 2009.
12. Liu Xiao-jun, Li Xian-feng, Chang Ying-xiang, Zhang Jian-gang. Chaos and Chaos Synchronism of the RikitakeTwo-Disk Dynamo. Fourth International Conference on Natural Computation, IEEE computer Society,DOI10.1109/ICNC.2008.706:613-617, 2012.
13. T. McMillen. The shape and dynamics of the Rikitake attractor. The Nonlinear Jour., 1, pp1-10, 1999.
14. Mohammad Javidi, Nemat Nyamorad, Numerical Chaotic Behavior of the Fractional Rikitake System, World Journal of Modelling and Simulation, Vol. 9 No. 2, pp. 120-129, 2013.
15. J. Llibre, M. Messias. Global dynamics of the Rikitake system. Physica D, 238, pp: 241-252, 2009.
16. C.-C. Chen, C.-Y. Tseng. A study of stochastic resonance in the periodically forced Rikitake dynamo. Terr. Atmos.Ocean. Sci., 18(4), pp: 671-680, 2007.
17. Ahmad Harb, Bassam Harb. Chaos control of third-order phase-locked loops using backstepping nonlinear

controller. Chaos, Solitons & Fractals, 20(4), 2004.

18. Ahmad Harb, Nabil Ayoub, Nonlinear Control of Chaotic Rikitake Two-Disk Dynamo, International Journal of Nonlinear Science ,Vol.15, No.1, pp.45-50, 2013.
19. U.E. Vincent, R. Guo, Finite-time synchronization for a class of chaotic and hyperchaotic systems via adaptive feedback controller, Physics Letters A 375 pp: 2322–2326, 2011.
20. Park, J.H., Ji, D.H., Won, S.C., Lee, S.M.: Adaptive. H∞ synchronization of unified chaotic systems. Mod. Phys. Lett. B 23, 1157–1169, 2009.
21. Jeong, S.C., Ji, D.H., Park, J.H., Won, S.C.: Adaptive synchronization for uncertain complex dynamical network using fuzzy disturbance observer. Nonlinear Dyn. 71, 223-234, 2013.
22. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modelling and control. IEEE Trans. Syst. Man Cybern., Part B, Cybern. 15, 116–132, 1995.
23. Feng, G.: A survey on analysis and design of model-based fuzzy control systems. IEEE Trans. Fuzzy Syst. 14, 676–697, 2006.
24. GHOLIPOUR, Y. "A REVIEW ON COMPARISION OF THE EFFECT OF I AND PI CONTROLLER ON A QUADRUPLE-TANK PROCESS."International Journal of Engineering Vol.6, No.01, pp. 13-16, 2015.
25. Wu, S.-J.: Affine TS-model-based fuzzy regulating/servo control design. Fuzzy Sets Syst. 158, 2288–2305, 2007.
26. David E. Goldberg, "Genetic algorithms in search, optimization, and machine learning", Addison-Wesley Pub. Co., 1989.
27. Genetic Algorithm and Direct Search Toolbox user's guide retrieved from: http://www.mathworks.com/access/helpdesk_r13/help/pdf_doc/gads/gads_tb.pdf
28. Randy L. Haupt, Sue Ellen Haupt." Practice Genetic Algorithm "Second Edition, A JOHN WILEY & SONS, INC., PUBLICATION, 2004.
29. Gholipour, Y. and Mahmood M., "Investigation stability of Rikitake system." Journal of Control Engineering and Technology 4.1, 2014.
30. Tsuneji Rikitake. Oscillations of a system of disk dynamos. Mathematical Proceedings of the Cambridge Philosophical Society, doi: 10.1017/S0305004100033223, Vol.54, pp. 89-105, 1958.
31. JOHN H. MATHEWS AND W. K. GARDNER_1968, "Field Reversals of "Paleomagnetic" Type in Coupled Disk Dynamos", U.S. NAVAL RESEARCHLABORATORY, Washington, D.C, 1968.
32. Gholipour Y., Shams E.M., "Introduction new combination of zero-order hold and first-order hold", Vol.5, No.2, pp.1269-1272, 2014.
33. Gholipour, Y., Ramezani A., Mola A. "Illustrate the Butterfly Effect on the Chaos Rikitake system", Bulletin of Electrical Engineering and Informatics, Vol.3, No.4, 2014.
34. Gholipour, Y., Mola, M. STABILIZATION OF CHAOS RIKITAKE SYSTEM BY USE OF FUZZY

CONTROLLER. SCIENCE INTERNATIONAL-LAHORE, Vol.27, No.1, pp.115-119, 2015.

35. Gholipour, Y., Chavooshi Zade, M.; “REPLACEMENT UNSTABLE TRANSMISSION ZEROS FOR A NON MINIMUM PHASE QUADRUPLE-TANK PROCESS”. SCIENCE INTERNATIONAL-LAHORE Vol.27 No.2, pp.1097-1100, 2015.